Fluid Mechanics
Prof. Subashisa Dutta
Department of Civil Engineering
Indian Institute of Technology-Guwahati

Lec 30: The Navier-Stokes Equation III

Good morning all of you. Today we are going to discuss on nebustic locations and its approximation for simple flow problems. We will discuss that part. As we discussed in the last class how we can derive the basic Bernoulli's equations from Navier-Stokes equations that what we did it Navier-Stokes equations from that we derived Euler equations then we have derived Bernoulli's equations. So that components also today I will recap it then we will go through these today topics which mostly I am following it the book of F M White fluid mechanics books for today lectures. So you can look at the similar line of derivations in F M White book.

So that way let me go through the basic things what we discussed that basically we are going to introduce velocity potentials okay. I will discuss more details. very interesting things, generations of rotationality. And then we will have a look at a simple solutions for incompressible viscous flow between a fixed and a moving plate.

Same incompressible viscous flow, we will look it for a due to the pressure gradients between two fixed plates that is which acute flow what we discuss it. Then we will discuss about incompressible viscous flow in cylinders that is what we will be discussing that. So basically let us start with Navier-Stokes equations which is long back about 200 years back. So the equations what we derive in last two classes we will go more detail about that. Before going that let me I just write down the basics equations is the Euler equations okay.

So if you look it when I talk about Euler equations you can understand it. It is for incompressibles and non-viscous or fixed or less flow. So this is the reasons we can apply it that means if you consider flow past a tall building okay a high rise buildings okay flow past tall building. So if I am looking for flow past or tall buildings like most of the Indian cities about today we can see these tall buildings. So if you consider a uniform stream flow we can get in these stream lines like this.

We can get a stream like this. But there will be there will be flow separations going to happen it. So if you look at this part of the flow okay if you exclude the near to the structures the flow outside of these structures the Euler equation is valid. But newer to structures as the vorticity is there, the turbulence behaviors are there as well as it has the flow separations that is what is not valid for this external flow. But beyond that you can see that the streamline patterns of this part I can always define through Euler equations which is very basic equations is rho Tv dt its acceleration is equal to rho g it is a vectors minus grade p.

that is what the Euler equations that is what is the Euler equations and this equation is quite valid for a flow past and tall building the reasons what we are quantifying beyond these reasons. Beyond these reasons that we can always apply Euler equations as well as we have a the derivations of the basic equations is the unsteady Bernoulli's equations also the steady Bernoulli's equation unsteady Bernoulli's equations and also the steady Bernoulli's equations. If you look at these equations still we have the in Euler equations in Euler equations still we have 3 unknowns u, v, w and p the pressures. So 4 unknown components are there 3 are the scalar velocity field 3 are the scalar velocity fields are there which is u, v, w and the pressures that is what the 4 unknowns are there. The basic idea comes that can you write these equations with a single scalar value okay that is the simple idea comes is that instead of looking at 3 scalar velocity component why we cannot write it with a simple a scalar functions that is what is the velocity potential function.

potential functions is similar to analogous to the electric field potentials and the current. So here we look at not the current we look at the velocity field that means we define it that the V can define as a scalar field of grade phi and the phi is the velocity potential functions phi is the velocity potential functions okay. So if I define that part okay so velocity this is what velocity potential functions okay single functions which is have a the variability functions of spaces time, functions of spaces time okay. It is not confined to two-dimensional as we did for the streamlines for the potential velocity potentials it is a three-dimensional also the unsteady part is there okay. So do not be confuse yourself.

So let us have x, y, z also that there is a three-dimensional functions in x, y, z as well as it has a time functions. So if I have a these components that means if I just equate it you need not to remember line by line or do not be confused that what I can write in scalar component a very simple way this. is equal to del phi by del x i del phi by del y j del phi by del z k. So it is just the expressions what I am putting it. just the expressions I am putting it this is what vectors form this is in a scalar components.

So what it indicates it is a very simple things u is equal to del phi by del x v small v the scalar components del phi by del y and w will be scalar component del phi by del z okay. So for this now the velocity potential functions what you have defined it is a single functions which is having a space and the time but using that single functions as its gradient the partial derivative with respect to x, y and z they are respectively representing you as u, v and w. So instead of solving u, v, w if I know these five functions I can solve these problems. That is the idea that instead of the three scalar variables I am targeting only one variables that is the velocity potential functions. Now if you look at that at which condition does it satisfy.

Thus if I define it the velocity is a gradient of phi, phi is a velocity potential function at which condition thus this is what justified it was. If you look at the very basic components if I velocity potentials. v equal to grade phi is the gradient of a scalar components will be justified okay or the reverse is also true that when you have v is equal to 0 okay. If other way around if this is the functions we will get it the cross pattern between del and the v is equal to 0. What does means see is that this condition is satisfied only these conditions when you have a irrotational flow okay.

So the reasons where we have a irrotational flow then those the reasons we can use the velocity potential functions to define our flow fields. and the velocity potential functions what we are defining it is a three-dimensional components okay x, y, z also it has the time component. But the conditions what we have the flow should be the irrotational flow field or the regions where the rotational activities merely or negligible components okay. So those the reasons of the fluid space we can apply the velocity potential functions. That is what will be give us advantage instead of taking 3 scalar component of u v w only the 5 the scalar components will be enough for us to solve the problem.

That is the advantage but the condition is that it should be irrotational flow. That means if I again coming back to the same flow first a tall building if I draw the two streamlines let I draw S1, S2 two streamlines if I draw the two streamlines S1 and S2 Then this is the streamlines which is the psi functions okay this is stream functions okay. The psi functions is a stream functions which is defined by a constant value of c1 and c2. So this is what we have the if you look at the velocity potential functions okay velocity potential functions which we define it v is equal to grade phi. So please look it as we derive it mathematically and also we prove it that if this is the conditions your del cross v call vectors of v that is what will be the that is what it means that irrotational flow okay.

So if I go for again for the same tall buildings and draw a two streamlines okay the psi equal to 1 psi equal to 2 then we can look it this is the streamline functions okay. So we can have a any constant value substitute to 1 and 2. So you need to find out a functions which is have a orthogonality that is what is the functions which is phi equal to c1, phi equal to c2, phi equal to 3. So this is the velocity potential functions. This is the the streamline functions.

This is what streamlines both of the things we call the flow net. More details I am not going here. So we need to draw the streamlines and the potential lines to show it that because just you interpreted the gradient of the velocity potential functions indicates as the velocity field that is very interesting part of here. So you can get the velocity field if you know these velocity potential functions but The same way if I can draw I can draw the streamlines I can draw the velocity potential line. The only things is difference here that the stream functions exist for two-dimensional cases okay.

So we will be drawing these flow nets streamlines and the equipotential lines for a two-dimensional irrotational flow. That is the conditions you can understand it because stream functions need a conditions that flow should be two-dimensionals and the velocity potential functions needs that the flow field should be irrotational. So both the conditions should be satisfied to have a the lines of streamlined as well as ecopotential lines or velocity potential line. So now if you look it as I draw drawn here these should have a orthogonality natures that is what we are discussing it. The streamlines and the potential lines they will have a orthogonality each others that means they will have a wherever they will be cross each others at that point they will have a 90 degree angle between them that is the orthogonality natures of the streamlines will get it.

Let me have a very simple way look at this relationship the orthogonality relationship it is not a big idea that you take a phi functions okay it is a line let me a two dimensional functions okay. So if you have a consider is phi is a constant and this line okay that is what along this velocity potential functions. functions that what we get it that by definitions that since it is a constant. So d phi equal to 0 and as we define it, it is a two-dimensional functions because for getting a relationship with streamlines. So that way if I have a d phi that is what will be defined d phi by dx that is what you know it basic chain rules okay for two independent variables of dx and dy you will get it that is what is equal to 0.

As you substitute the values and if you want to know it this is let me x and y we are looking it what is a dy dx the slope of this part if I just rearrange it. I will get a dy by dx okay that is what will be equal to minus u by v this is for the phi equal to constant this minus u and v if you look at that definition of the stream functions we can get it just reverse 1 by dy dx. of the stream functions is a constant. That is what the slope of these two products if it is minus 1 as you know it that means two lines are interacting meeting at 90 degree angles or they follow orthogonality behaviors. So this is what from basic the relationship between the streamlines and the velocity potential functions we can get it this if you have a velocity potential function c1 and c2 and the streamlines will have a 90 degree to that.

So this is what the streamlines. this is what streamline will have a psi maybe have a c3 psi equal to the c4. So that is the functional variability we getting for streamlines and the velocity potential functions. Many of the times when solve the equations or try to understand the flow patterns first you draw the streamlines and if it is a 2 dimensional flow you can easily draw the velocity potential functions which will be indicate us the velocity how the velocity fields are very good that is what we can always interpreted it. Now if you look at very interesting part which I can relate it with very basic equations what you know it is the Newton's first law of motions is you know it if you look at that Newton's laws first law of motions okay you know with the three laws okay.

If you can remember the first law of motions what it says that okay that is the same concept what we will talk about rotationality okay the generations of rotationalities. Just try to understand it is what is the Newton's first law says that Newton's first law says that a body is remains at rest or its moves at a uniform speed in a straight line unless otherwise you all-bundle force acts on them. okay. So again I have to repeat it as if you try to understand it if a body okay that is what is moving with a uniform velocity it will move in a straight line unless otherwise there is external unbalanced force act on that.

The same concept is here. The concept is same what is there in Newton's the first law of motions which talk about when a body is moving it if you are not giving any external force it will not be accelerated de-accelerator or it can change the directions of motions. The same concept is here instead of the laws of motions here we talk about the rotations that means if the flow field is irrotational okay flow field is irrotational. irrotational that means let for examples you have a uniform flows let us you talk about there is a uniform flow okay the same velocity v the constant velocity is proving it this exactly same law okay that is what is you should correlate it is not much difference between fluid mechanics and solid mechanics. here we do not visualize the body movements or object movements. You have to have a conceptually understand it.

For examples that you have a wind movements having the uniform velocities. So when you have the uniform velocities you will not have any the nature of irrotationality is not change it unless otherwise you put a object here okay put a object here which will create a boundary layers okay low viscosity zones create the vorticities then it will change the irrotational properties it will go for the rotational behavior the same concept what we call the generalized generation of rotationality. please try to understand it the same concept the Newton's first laws of bosons which you could have learn it at the class 10th levels. Same concept we are looking it when you go for whether you have a wind flow uniform wind flow you will have a irrotational field unless otherwise you put a object create a viscous stress components as you reduce the velocity as you get this object and the velocity reductions those regions will be create the viscous regions then that is the reasons it will be start the rotationalities that is what you try to understand it okay.

just try to understand it okay. That means a fluid is initially irrotational maybe may become rotationals okay only if there are the four conditions okay. The first condition is that as I discuss it which we can easily understand it that if there is a significant Vixus force induced by jet, wake and solid boundaries that is what I have incorporated here. then the flow will be change it chance goes for the rotationalities. And as it goes for rotationalities and the viscous dominance is there we cannot use the Bernoulli's equations okay that is what our so popular so familiar equations it is no use for us when you talk about where the viscous dominance is there. So that is what talk it here that we cannot use these equations when you have the flow changes for rotationalities and the viscous components dominates are there.

Same way if you look it that is what is represented here. So if you have the aerofoils like this if you have a aerofoil and you have a uniform flow it will be irrotational okay but as soon as it touches it creates the laminar boundary layers below it can have a turbulent the flow separations and they then you can have a wake flow okay. you can just look it how the behavior is happening it this is what subsonic levels but in case of supersonic levels this is what the supersonic approach you will have a shock wave okay we have a shock waves that is what it it also changes entropy gradients if you have a curved shock waves that is what is here okay that is the second conditions. First conditions if you have a solid boundary, jack and wax then the flow will be change it to the rotational. Same way if you have a the shock waves because of supersonic flows also will have the entropy gradients will get it that is what will make it flow rotationals okay and if you look it that way.

Also we can look at whenever we have the density gradients that is caused because of uneven heating. So that is what it happens it. So that what will be cause the density gradients will get a density gradient because of stratifications that is what because of uneven heating or because of earth rotations will get it non-linear effects. So that is the four conditions are necessary to change the flow field which is initially irrotational to the rotational field. This is what is equivalent to the Newton's the first laws of motions which talks about object motions.

Here we talk about the fluid conditions changes from irrotational to the rotational field. Any questions? Can you explain the shock waves formation against More details I am not going here because this course we are not talking about compressible flow. I that is the reasons I am not going detail but if you look it in case of compressible when you have a supersonic and compressible flow you will get a shock wave patterns okay because anyway the course is for undergraduate levels I will not go to that level. Okay the questions what you have I think if I understood it that if I have a object okay and I get the boundary layer formations okay. just outside of the boundary layers I will have the flow a close to the frictionless flow okay.

This is close to the frictionless that is the reasons we will discuss in the next chapters how have will be linked between boundary layers and Euler methods to solve these problems instead of looking these Navier-Stokes equation solvers that is what we will look it but these are the reasons The outside of this part will show you the viscous is not dominated. That is the reasons we can use Euler equations to solve these problems. or if you know the streamlines you can just apply the Bernoulli's equations to solve the problems. So but inside this for the this is the boundary layers where we will discuss more how to link these two conditions instead of solving the Navier-Stokes equations we can solve these boundary layers and the Euler equations to solve external flow field. But if you talk about any internal flow field internal flow for example of pipe flow.

So you can understand it is totally viscous dominancy will be there. So the here we will not have a region which shows us the flow is irrotational okay because when you have a pipe flow it is a total viscous dominations are there and the flow field this irrotational activities are there okay as vertex and formations even if in a case of the laminar flow also. So internal flow will not have a much Euler equations but let me give a simple examples as you put it that concept. Many of the you I think the students have solved these very basic equations to find out problems using the Bernoulli's equations that if there is a tank and there is a outlet here and it is having height h okay.

We need to compute what is the velocity here okay. Without looking anything we know it V is equal to square root of g h okay. So this is the problems you could have solved in the 10th class levels or the 12th class level But how we did it that we do not know it. How we apply these things if you visualize the flow field. For examples if I draw the streamlines okay the flow near to the surface will be viscous dominant.

The streamlines will come like this. Streamlines will come like this. Streamlines will come like this. streamlines can start from here. Now we can say that I can apply take a streamlines okay take a 1 and 2 I can apply the Bernoulli's equations and get it the same format okay. The use of Bernoulli's equations give me the same the relationship that v is equal to if you look at this part if I apply this Bernoulli's equations that means I am telling it this part that same way I can get it v equal to rho Just apply the Bernoulli's equation at 1 and 2 you know the velocity 0 at this point you have a pressure is atmospheric at 1 and also pressure is atmospheric at the 2.

So that way you can just apply it you will get it the V equal to 2gh but try to remember it that there is a rotational activities viscous stress components are there. Near to the surface you will have the boundary layer formations. Near to the surface you will have a boundary layer formations. The boundary layer formations are there. So using Bernoulli's equations we are just not trying to understand it the microscopically what is happening it.

We are just try to look it a macroscopic scale that I am getting the velocities. But is it what velocity I am getting it. is it average velocity it is what is that because there is a formations of boundary layers there is a rotational activities as the flow is comes it if you take the field. So more detail we will discuss it that is what is objective of these course to not to get v equal to square root of 2gh which it is not necessary to understand the fluid mechanics to solve these problems. But you try to understand what type of stream functions are there and if these are the stream functions what should be my this could be my potential functions okay this could be my potential function.

This could be my potential functions. So you should try to understand it very detailed way how what should be the streamlines what with the velocity potential functions where we expected it there is a formations of boundary layers because of no slip conditions of net to the structures. that what it will be retracted that is what is here the conditions of boundary layers will be there and that what is will be prevailed with these four conditions what we are talking about that you should remember just for example of Newton's laws of first laws of motions the same concept you are talking about here that fluid is changes from irrotational to rotational if any of these four conditions prevails. That is the guidelines when you want to draw any streamlines, you want to draw a equipotential lines you should know it whether these things are there or not. And second things very easy things for us because when you use this velocity potential functions if I am using this velocity potential functions then if I am using the velocity potential functions is instead of u vw the scalar components I just use velocity potential functions okay and that is what will be easy for me if you can refer to FM White book even if unsteady Bernoulli's equations we can write it in terms of the phi okay instead of you can write unsteady Bernoulli's equations okay equations in terms of phi that is not a big you can just refer to that book. But what is the advantage that when you define V equal to grade phi okay and your mass conservation equations or continuity equations is that the velocity divergence that is what is give a Laplace operator of the phi, this is what.

So basically your mass conservation equations when you write in terms of velocity potential functions is comes out to be a Laplace's function of phi is a linear functions which we can easily solve it. If it is a linear functions of phi if I getting it okay it is maybe the second order but the linear functions which can we can easily solve it. That is the two advantage we have when you write in terms of velocity potentials. instead of solving these 3 scalar components we define through a single functions which is the velocity potential function. Our continuity equations becomes a Laplacian equations of Laplace equations of the phi which is very easy to solve it as well as we can write unsteady Bernoulli's equations.

That is the strength of velocity potential functions and the stream functions what we discussed earlier. I think that is what it will be a consider it is a key points when you try to solve a many problems complex problems. Even if you solve the problems using computational fluid dynamics tools but always you can draw the streamlines you can draw the equipotential lines then you try to understand it. where the behaviors are what okay that is what you can just look it how the behavior patterns are there okay. So the steam functions are used even after CFD solvers to understand it the flow patterns.

Now coming back to doing some simple approximations using Navier-Stokes equations to get simple solutions of between a fixed and moving plate viscous flow if you have we called cute flow okay. Conditions is that I have a fixed plate okay. So means the velocity is 0. I have a moving plate V.

I am looking at what would be the velocity distributions okay. I think these problems we started it first chapter of the fluid mechanics is a Newton's laws of viscosity. the same concept we are talking about here. If a plate is moving with velocity v what should be the velocity distributions, what should be the shear stress distributions that is what we are looking at for viscous flow. rho is there, mu is there all are constants. To simplify the problems we can take it the central lines is a axis okay this is the x axis this is the y directions this will be the plus h coordinates this is minus h okay and I have the plate which is moving at v and I am trying to locate what could be the velocity distributions for viscous incompressible flow.

That is my point. If you see that what are the things we are neglecting which justified for us. Like when you talk about a horizontal plate movements, so we need not to take the gravity flow. So that means we can neglect it.

We can neglect the gravity component. That is what is quite feasible. See if the plates moving in horizontals, no doubt we can ignore or you can neglect this gravity flow. But just try to understand it if you have the vertical motions, you cannot do that. that way you can look at how you are solving the problems. So you have neglecting the gravity components. Second thing is that since this is a free movement of the velocity v so you can neglect that the pressure gradient part okay there is no significant pressure gradient part will be there okay.

So del p by del x also can be neglected. Another term is there the fully developed flow I think these things we discuss in the pipe flow the same things I will tell it. We making the velocity v such a way that after certain times the velocity distributions are not changing it more or less remain the same that the conditions we call fully developed flow. fully developed flow so basically it should be far away from the entrance okay. This is what because it is a long plate so we can see that only this axial flow is there your the velocity component in y directions and the z directions both will be the 0 okay.

neglect the fresher components as the part is there. So always you when you look the problems simplify the what are the things we can simplify because if you look at that Navier-Stokes equations is nonlinear equations. So as you will have simplified it that nonlinear partial differential equations we can convert to a simple ordinary differential equations which we can integrate it to get the solutions that is the strategy always. as a fluid mechanics specialist always we look it which are the components are dominated which are the flow fields we can neglect it. For examples in this case it is only axial flow so on this x direction so other components like V and W is 0 the pressure gradients are 0 since the flow in a horizontal directions the gravity components are not it can be really neglected. So if you look at that these assumptions are more important as compared to substitute these components into the acoustic equations then you get the ordinary differential equations and do a integrations apply the boundary conditions to get the functions that is what the steps.

Now if you look at the same components if I look at as velocity v and I have the plate is moving like this okay. The first equations what we look at is the continuity equations. Continuity okay. Velocity divergence is equal to 0 dU by dx dV by dy plus dW by dz is equal to 0.

So this is 0, this is 0. So that means y is a only function of y is u is only functions of y that is all. So this is what is justified from this velocity divergence field that as we know it that other components are 0. So let us not consider z direction it is a too wide. So that means only the velocity will have a functions we will expect to the u velocity components will have along these y directions that is what is what we are considering it. Now if you apply this Navier-Stokes equations okay that is the part if I look it the Navier-Stokes equations if I apply it if I just rewrite it here.

okay. So starting from it is a steady flow that is what again I am highlighting it because it is a fully developed case okay that is a steady flow. So if it that if you write the navier-strict equations in x directions neglecting some of the components because of steady flow and all you will get rho u del u by del x v rho u by rho y z components I am not writing it because it is not having functions minus del p by del x plus rho gx mu times of always you keep it your interpreted the things okay which we should make it 0 okay that is the idea about this strategy of Navier-Striegge equations when you try to get analytical solutions always your strategy is that which are the components are 0 okay. No doubt this is already 0 you say that gravity does not matter pressure gradient is 0 these terms are 0 okay. So if you look it that way only this terms is left it this components 0 as it is a only y functions that is what is from this we get it del square by u by del y square or we can write in terms now only t y square is 0.

So you can get a functions a linear functions with y. that is very simple things linear functions of y that is what you see that is it looks very complex that we have used non-linear equations like Navier-Stokes equations we use the continuity equations but end of the day we are coming back to a simple ODE equations where we can get analytical solutions which is a having a two constants c1 and c2 are two constant and these two constant we can get it applying this boundary conditions as you know it that when y equal to the minus h the velocity is 0, the u component will be 0, y equal to plus h the small u velocity is v. So if you substitute these conditions then you will get this the velocity field. that is what you will get this velocity field just to I sketch it these components that what I am getting it after solving these problems this is a fixed plate this is fixed this is the plate. moving with velocity v which is as equivalent to well-known experiments is Newton's laws of viscosity experiments with a plate moving. If you look at the viscous flow it will have a linear velocity okay at y equal to 0 we will have a you will have v by 2 that is what you can look it y equal to substitute 0 you will be v by 2.

So you have a linear velocity for profiles you will have the linear velocity profiles what you get it. So if you look at that it is easy to get it from experiment what is necessary but this is what the analytical solutions we get it with a series of approximations we have done it for the specific problems. Now if you will go for a next problems which is another simple problems okay that there is two plates are there okay two plates are there it is not a pipe flow it is a plate okay just like you can see that some sort of lubricants are moving it within a two plates okay both are the fixed but they are flow is moving because of the pressure gradient and as the pressure decreases the flow is moving because of the pressure gradient. Due to the pressure gradient the flow is moving that part we are looking it for a incompressibles and the viscous flow rho and mu constants and the same coordinating systems like x, y, minus h and plus h we try to solve it what should be the velocity field. So, in this case both the plates are fixed the flow is happening due to the pressure gradient what should be the velocity field.

So, if you look at that same way if you can approximate it okay you will get it the very basic things that in x momentum equations okay I am not deriving step by steps you always can follow this fm white book to get it like last examples what I did it cancelling out all the terms which is which are the 0 the same way if you do it I will get this part okay that means I will get mu del square u by del y square is equal to minus grade P by grade x. So this is what the x momentum equations. If I apply for y and z momentum equations where the v and w is 0, the same conditions.

Here v, w all are 0 values. It is only the axial flow. It is half negative. So because of that you will have a dP by special derivative of P in y and z directions will be 0. So looking this P is only a function of x only that is all. As P is only a function variabilities and x directions that is what indicating here if I substitutes that okay there is a few of the steps we have not discussed line by line. If I substitute that and dP by dx is if I consider is a constants okay and then if I just look at the solutions of these conditions we will get it the u is a functions of the gradient and y square y and the c1 c2 two constants are there. So if you look at these two constants also I can get it because when y equal to minus h here the u velocity is equal to 0 at this point when y equal to the plus h the u is equal to 0 okay.

So if you substitute that you will get a c1 and c2 values so you will get a paraboloid functions okay. which is similar to the pipe flow. Here I am talking about flow through the plates. We are getting this the flow patterns which comes like this and always you can find out what will be the u maximum here.

That is what you can get it just substitute y equal to 0. You can get the u maximum and you can just integrate the u the velocity functions okay from minus h to the plus h dy into length okay which is a unit value you can get the discharge. we can get the discharge. You see if I know the velocities if you just integrated what this and the unit value if I consider the perpendicular to this plane that is what I will get the discharge. So same way if I know the velocity distributions also I can get the wall shear stress. I can get the wall shear stress those are examples we have but today let us complete this part with this derivation. Thank you. Thank you.

Fluid Mechanics
Prof. Subashisa Dutta
Department of Civil Engineering
Indian Institute of Technology-Guwahat

Lec 31: Approximate solutions of Navier Stokes Equation: Bounary Layer Approximation

Good morning all of you. Today we are going to discuss about approximation solutions of Navier-Stokes equations. That is what we earlier you started. to how to approximate the Navier-Stokes equation for simple problems flow through two plates with movable plate with pressure gradient. Same way we can look it for solutions of Navier-Stokes equations for pipe flow through a concentrated cycles circles we can do that. But today we will solve to problems, mathematical problems, then we will discuss about the boundary layer approximations for Navier-Stokes equations.

That is what today very detail I will talk about boundary layer approximations, how we do it for boundary layer problems. That is the problems what I will introduce to you. I can look at the mostly we are following Sinzel Zimbala books as well as F. M.

White for this part of the lectures. Basically what I will talk about these most of the books I will come back to that. So we will talk about how we do this boundary layer approach equations and also boundary layer approach equations. Before that Again we have to solve two problems maybe next class we will talk about different thickness like displacement thickness, momentum thickness and also the turbulent plate boundary layers that is what we will discuss it later on part. Let me I go to solve very simple problems.

The problem is that there is a steady again I am emphasizing two-dimensionals incompressible velocity field is given to us. We have to compute a functions pressure is a function of x and y okay. So if you look at that the velocity field what is given to us which is having the scalar component in x directions Ax b, this is u functions and you have minus Ay plus Cx j functions okay. So this is the u component and this is the v component. okay.

So this is what we have the functions of u b the scalar components. You have to check first what are the assumptions are there. If you look at that both the u and v does not have any time component. So it is a steady flow. You just to have to verify it.

So it is a steady flow as the u and v scalar component does not have a time component. dependency so is a steady flow. And u and v does not have a functions of z so the flow is 2 dimensional. So this understanding you should have. So as u and v does not have any dependency function in the z directions so we have the 2 dimensional flow.

And this rho is the constant as it has given for us the density is a constant for incompressible flow. So these things are given to us. Now we are looking it whether what is the pressure field we have to compute it which will be a function of as a 2-dimensional flow, steady 2-dimensional incompressible flow. So we will have a functions of x and y which is there. With us what is there? because two-dimensional incompressible flow we have a continuity equations.

You have continuity equations for two-dimensional flow you know it del u by del x plus del v by del y that should be equal to 0 that is then we have a the Navier-Stokes equations in two directions x and y direction. Z I would not need it because it is a two-dimensional flow field, two-dimensional flow field. So with us we have a continuity equations, we have a Navier-Stokes equations which are linear momentum equations in x and y directions that these are three equations we will put it to first. So continuity equations we will use it to confirm that whether this equates the velocity field what is given it a functions of x and y do they satisfied or not. That is what is that flow is steady, incompressible and two-dimensional in x and the gravity does not act in either in x and y direction.

So another assumptions we have put it so that to neglecting this the gravity components okay that is the assumptions if you can look it. Now if you look it first to check it does this velocity field satisfy the continuity equations. If you satisfy then this is the basic velocity field for for which we can derive the pressure field. If the velocity field does not satisfy the continuity equation, we should not find out the pressure field for that. So as the flow fluid, we have a continuity equation, so we have an eviscerative equation.

We are looking at doing the continuous equations we are looking it which is incompressible continuity equations we are using is to satisfy whether the velocity field what is given it really for the fluid flow case or not. So you can do a simple partial derivative of u field del u by del x if you look it is a del u del v by del y it will be minus A just by just inspections you can locate that what is coming it. So this way you can always find out through just looking these components that del u by del x is A and del v small v by del y is minus a. So combinations of these will be 0. That is what it is satisfying.

That means the velocity field whatever is given it that is satisfying the continuity equation. So we can determine the pressure field that is what we can define the pressure field. If continuity equations are not satisfied we could stop here because it is not a given field could not be a physically possible okay that is try to understand it. So what I want to emphasize is that you look at this is not mathematics problems, it is the problems of fluid flow. So first foremost requirement for us to get the pressure field the continuity equation should be satisfied it.

If not then in order to compute the pressure field because it does not refrigerant the flow of equations that is that flow field. So that is the idea you should understand it. The same way if you look it I am just go for the pressure field calculations. As I said it we have a two momentum equations. We have dropped the gravity components.

Again I can write it two-dimensional even if it is here two-dimensional Navier-Stokes equations let me I write for in the x directions okay that is what you can have a practice okay. we remember it is a steady. So we do not have a local accelerations component. It is a steady flow. So there is and it is a 2-dimensional.

So z components is not there. That means W is a 0. So anything by del by del z or del square by del z square all are 0. So that is very basic things you should know it as soon as is a two-dimensional things that is what will be help us will help us to write Navier-Stokes equations in a simple form. So we have only convective accelerations in two directions.

So first x directions I am writing it. So this is v del u by del y that is a convective acceleration part rho gx minus grade P by grade x plus mu times of Laplace operators that is what we are expanding here. That is what you have to have a trick to how to expand Navier-Stokes equations. You can expand it, look at the local acceleration term, okay. There is a gravity force component, the pressure force component and viscous force.

This is 0. Anyway, we made it. There is no component of gx. So what I have? I have the second derivative of second partial derivative of v you would expect to x and y all will be the 0. So just looking the functions okay. So only I have these two terms I have only these two terms.

So as I have the two terms if I just rearrange it I will get it del P by del x is equal to rho times of this is what coming from the convective acceleration terms as you do the partial derivatives multiplied with v partial derivatives with x by y multiplied with v will get it a square x minus Av. Same way if you apply the moment of equations in the y directions I will get this component. So just same way I will get the component. This component anyway 0 and this component is 0. So again you will get it a partial derivative of pressure field with respect to x directions we have a functions we have a.

So from these we have to derive Pxy. So the basically we are looking at what is a function of Pxy the pressure field that is what you are looking at. So, here we do a very simple calculations we do it with integrations finding out the integration constants substitutes or methods what we in generally do in mathematics subject. The as you know the partial derivative of pressures of in x direction y direction how to estimate the pressure p x y it is a simple integrations and putting this concept. The for examples first you can just find out the pressure field should not be a smooth functions should not have any sudden discontinuity that is one means either of the P or derivative of P they do not have a to any discontinuity functions that is what you can looking the inspections you can find out.

Now if you look it that requires to you can differentiate it also you can cross differentiate it like to know it whether this cross differentiate that is what it is giving a conditions for the smooth functions okay is equal to if it is equal to the del x by del x del P by del y okay. If this is satisfied that means function is a smooth functions. There is no sudden discontinuity okay.

This is a partly it is mathematics part but please when you go for a partial differential equation form of Navier-Stokes equations looking for exact solutions always you will have look at the mathematics part more as compared to the fluid mechanics okay that is the difference. But I do believe it that is what you have to look at if you are looking at function p x y So as you have to look it whether there should not be sudden discontinuity.

So you have to change the order of differentials and then cross differentiate it if they are satisfying that mean velocity satisfy the steady two-dimensional incompressible Navier-Stokes equations. Now I need to compute the pressure field. which is again I will get that it is all the mathematical steps used to get it the functions. So as I know the dP by dy as a functions. So if I integrate it I will get the Pxy functions.

If I just integrated that I will get a Py functions with a function of the constant can be defined as a gs okay. So that gs you can just do a partial derivative of these functions then you can get it these functions this way and if you substitute gx value finally you are getting a functions looks like a having second order term x square y square x and y and a, b, c are the constants and you have additional C1 constant. So if you look it we simple if a velocity field is given it to estimate the pressure field we have to go analytically derive the pressure field using these two equations with us. The continuity again I am just revising you to that and Navier-Stokes equations. This is two equations three equations we are using for this is one, this for the two-dimensional case we have a two, three equations we are using it for a given velocity field we can get a pressure functions.

But we remember it, it is not like mathematics that we are using it, we are satisfying the conditions like whether the Pxy is a smooth functions or not. we are satisfying with a continuity equations to know it whether it is a velocity field what is given it does it satisfy the continuity equations. If it does not satisfy the continuity equations you need not to estimate the pressure field. This is not representing the flow field. There is no meaning if the velocity is not representing the flow field in a continuity equations then we need not to go for estimating the pressure field.

That is the difference between mathematicians and the fluid experts which look at that first the continuity equation satisfaction should be there then followed by we look at the Navier stoke equations to determine the pressure field. So that is the very easy problems what I have solved it but you try to look it how complex functions can be given it to determine the pressure field. Now second problems just to demonstrate it we do not require only the velocity field and pressure field. Many of the times we need to know it what is wall shear stress, what is stream functions for a particular flow field, vorticity, velocity potential and average velocity. So we are taking the same problems that there is a fixed plate okay the flow is going because of the pressure gradient because of del P by del x this is the pressure gradient and we have a x okay these are two plates are there which we in the last class we solved it pressure gradient is there and you have the velocity v and you have a y this is minus h this is plus h.